Pricing Electricity Forwards using the Real Option Theory

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29 February 2008

Abstract

Electricity Markets are liberalised in many countries throughout the world. Liberalised in the sense that the wholesale electricity prices are no longer fixed by government or a regulatory body, but rather determined by the law of supply and demand. Although the electricity market is new, the contracts traded over the counter and on exchange range from vanilla swaps to exotic options and other more complex contracts compared to capital markets. Nevertheless, there is no standard methodology for pricing such contracts as is the case of products in financial markets, where closed-form solutions of Black and Scholes type are commonly used.

There has been a quite substantial research aiming to produce a benchmark pricing methodology for forward electricity contracts. To-date, there are three main schools of thought. The first one essentially applies the risk-neutral valuation approach (with some adjustment) to price electricity contracts. The second one uses simple actuarial principles based on expectation of future prices and the concept of certainty equivalent of contingent claims. The third one considers an equilibrium approach where market players are maximising the expected utility of their consumption through time, subject to constraints involving conversion of energy sources into electricity.

From this perspective, the forward pricing problem based on spot electricity prices remains unsolved because the underlying asset (electricity) is not storable (economically) and cannot be traded from one period to the next. The theory of trading claims on non-tradeable assets (real options) is seen to have a natural application to electricity.

In this paper, we first examine the characteristics of the electricity pool prices and look at the relationship between the spot price average and the forward price. We then apply the theory of trading claims on non-tradeable assets (real options) to electricity. The resulting forward price depends on the volume underlying the contract, the spot price weekly averages for previous periods, the market price of risk and the interest rate.

In conclusion, this paper sets up a new methodology for pricing electricity forwards, where the resulting prices are more intuitive in capturing the market parameters as well as the characteristics of the underlying contract.

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1 Introduction

Even though the structure of liberalised electricity markets might be different from one country to the other, there are common features that make the study of one particular market useful in the understanding of the others. Moreover, the standard features of derivatives across all financial markets renders the pricing in one market relevant to the others. In this paper, we consider the Australian National Electricity Market (NEM) as a study framework. However, the development can readily be applicable in the context of other markets.

In this introduction, we provide a short description of the Australian National Electricity Market (NEM). Then we discuss different categories of market participants and their risk profiles. Finally, we explain how the forward market is used by these participants to mitigate their risk.

1.1 The Australian National Electricity Market (NEM)

The **NEM** started operation on the 13 December 1998. It is now comprised of electricity industries in Queensland, New South Wales, Victoria, South Australia and Tasmania which form one competitive wholesale pool. In 2006, the NEM oversaw the electricity production, network transmission and trading functions of 180,000 GWh per annum, an industry with annual turnover of over $6 billion. This is comprised of private businesses in Victoria and South Australia, and state-owned corporations in NSW and Queensland. The NEM is governed by the National Electricity Code which, among other goals, ensures competition among power suppliers in the multi-state service area in an effort to reduce overall energy costs for consumers and end users. **NEMMCO** (National Electricity Market Managing Company) is in charge of operating and administering the market according to the code. For more in-depth discussion about the NEM, we refer the reader to AFMA (2005).

1.2 Risk in the NEM and the Role of Forward Contracts

The NEM is a highly volatile market in which price-setting is ultimately driven by the law of supply and demand. As a result, occasional price spikes occur due to periods of unusual high demand, network transmission constraints, unscheduled outages of generation and the generators bidding behaviour.

The volatile nature of this market presents a significant financial risk to market participants such as generators, retailers, traders, consumers and market network service providers.

The magnitude of risk in the NEM maybe illustrated by considering the retail sector. Retailers typically set prices and commit forward sales to their customers for 2 year periods, guaranteeing a fixed electricity price for the period of the 2 year contract. The retailer sources this commodity (energy) by buying electricity from the NEMMCO on a spot half-hourly basis, resulting in a mismatch between their obligations which are fixed in price, and their funding, which is sourced and impacted by half-hourly price movements.

To illustrate the financial impact to a retailer, let us take the example of a corporate customer who runs an Aluminium smelter, whose needed electricity requirements (100MW) have been purchased through forward buying contracts set at $50 per megawatt hour (MWh). If the spot price was to spike to $5000/MWh for 3 consecutive hours, then the consideration paid to the retailer would be $50 x 100 x 3 or $15,000 (contract price x MW x hours), while the funding paid by the retailer would be $1,500,000 ($5,000 x 100 x 3); a **financial loss of $1.485 million**!

To mitigate the financial risk retailers use wholesale forward contracts, similar in characteristic to the contracts they sell to their own customers, except that the players are retailers (the buyer) and the electricity generators (the sellers), and the payoff is structured differently. The retailer's aim is to hedge against the sold fixed contracts to its customers, by funding the obligations with a series of wholesale fixed contracts bought from the generators, at a price that is lower than prices sold. Ideally the generators would be selling contracts to retailers at prices that are in excess to the costs of generation, and in turn, the retailers are selling fixed contracts to their customers that are greater than their process paid for purchases from the generators. This process is illustrated in (Fig.1) where the retailer pays the pool price to NEMMCO which then pays it to the generator. In the existence of a forward price, the net cashflow from the retailer to the generator is the difference between forward price and the pool price (positive or negative). The customer would typically pay a fixed price that is higher than the forward price.

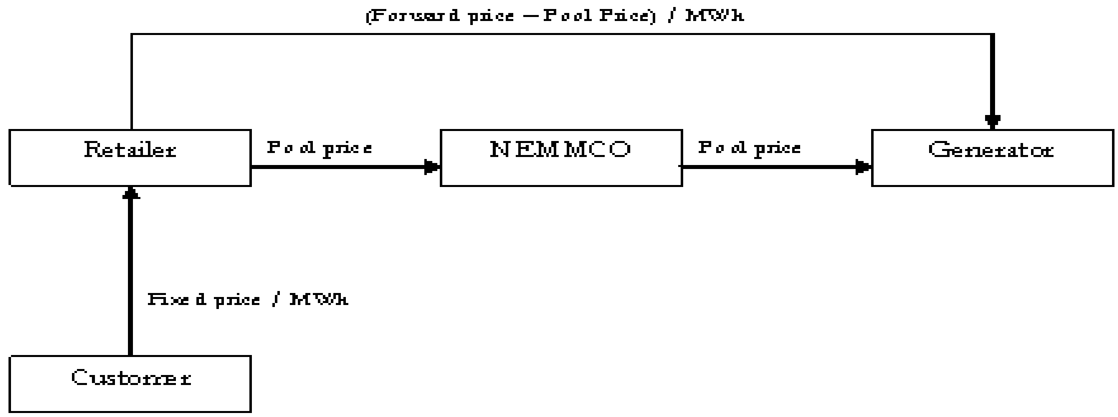

Figure 1: Typical swap transaction in the NEM

As far as the generator goes, if it gets dispatched by the pool, will earn the pool price, hence is exposed to low pool prices. A retailer, on the other hand, has committed sales to its customers at predetermined fixed prices. It buys electricity from the pool so has exposure to high pool prices.

This natural exposure to the different extremities of pool prices creates an opportunity for retailers and generators to set up a forward contract where the retailer pays fixed price to the generator in exchange of the pool price. The generator is then guaranteed a revenue (higher than the cost) of generating electricity, and the retailer is guaranteed a fixed cost of purchasing electricity which would be set to a value lower than the fixed price of selling electricity to its customers.

It might seem that with such transaction, both the retailer and the generator are completely hedged against price risk. In reality, this is not the case since the series of cashflows of difference payments exchanged between the counterparties depend on the volume underlying the forward contract. The retailer is committed to deliver electricity to customers for a certain future period, but does not know with certainty what volume its customers will consume in the future as this depends on several factors like weather. Therefore, the retailer is left with **volume risk** that can, however, be reduced by a more accurate forecast of load, based on weather and history of customer consumption.

On the other hand, the generator can still incur risk under such a contract if it did not get dispatched at the pool price because of another competitive bid submitted to NEMMCO or because of one of its units failure. It is still obliged to pay out to the counterparty under the contract and can result in a serious cashflow risk. This situation is exacerbated if one the units is big enough to cause significant reduction in market supply, which would lead to high prices.

1.3 Objectives and Contribution

Besides the non-storability characteristic, electricity is also non-tradeable, in the sense that once it is purchased, it should immediately be consumed and cannot be resold thereafter. Considering these two characteristics of electricity, one can think of the real option theory where the underlying is also non-tradeable and non-storable. For example, a mining project, once it goes ahead, the process is not reversible and the payoff cannot be replicated by trading in market securities.

The key aim of this paper is to develop a novel framework for pricing electricity forwards based on the real option theory. We attempt to answer the question about the forward premia from a perspective that takes into account the non-storability and the non-tradeablity of electricity. In addition, we also provide valuable insights into the behaviour of the Australian electricity pool prices by investigating key stylised facts and modelling the pool price average dynamics.

The paper is set out as follows. In the next section, a brief review of the literature is provided. Then, a detailed definition of electricity forward contracts is given, with an account for the relationship between forward prices and spot price averages. Section 4 provides an analysis of electricity pool prices, and proposes a model for the pool price weekly average using autoregressive models AR(3) for peak and off-peak periods. In section 5, we derive the forward prices using the real option theory. Prices for contracts on one period ahead are obtained in a closed form solution. For the general case, where the contract spans several periods in the future, a recursive formula is given instead. The final section allows for some concluding remarks.

2 Literature Review of Electricity Forwards

There is a clear relationship between forward and spot prices. This has been the subject of an extensive finance literature over the last three decades. In commodity pricing, many works including Hazuka (1984), French (1986), Fama and French (1987), study how accurately forward prices can be used in forecasting future spot prices. The literature on the pricing of forward contracts has historically focused on the arbitrage relation between spot prices and forward prices. There are two primary types of models that appear in this literature. The first is the standard no-arbitrage or cost of carry model, where an investor can synthesize a forward contract by taking a long position in the underlying asset and holding it until the contract expiration date. If the forward price does not equal the price of the replicating portfolio, then arbitrage profits are possible. Thus, the forward price is linked directly to the current spot price.

The classical literature on the cost-of-storage or cost-of-carry model includes Working (1948), Brennan (1958), Tesler (1958), and many others. This approach was applied to electricity by Schwartz (1997) for one, two and three factor mean reverting process, and Deng (2000) with mean reverting stochastic volatility process with two types of jumps. Hinz (2004) applies the known Heath-Jarrow-Morton framework to model forward electricity curve. He chooses 1 MWh in front of delivery as domestic currency unit, and with this choice all power forward prices finish at one, and therefore their dynamics are described utilizing the toolkit of interest rate theory. It is important to note that the no-arbitrage argument underlying this model relies on the ability of an arbitrageur to take a position in the underlying asset and hold it until the contract expiration date. **Since electricity is essentially not storable, however, the cost-of-carry model cannot be applied directly to electricity forward prices.**

Another feature missing in the literature is the dependence of the forward price on the volume of electricity contracted. The usual relationship $F(t,T) = (S_t + U)e^{(r-y)(T-t)}$ does not include the contract underlying volume. However a generator who contracts 1000 MW in 2008 bears more risk than the one contracting 10 MW. Therefore, importantly, the forward price should include a premium for the underlying volume.

The second general approach used in the literature to model forward prices is based on equilibrium considerations. Bessembinder and Lemmon (2002) develop an equilibrium model of electricity spot and forward prices in a production economy where they find that increases in forecasted demand have a strong positive effect on forward premia. Routledge, Seppi and Spatt (2001) model a competitive rational expectations electricity market using a setting where storable commodity such a as gas can be converted into electricity.

Another recent approach consists of modelling the primary system drivers that influence price setting: supply and demand. Eydeland and Wolyniec (2002) introduce hybrid models in which power price forecasts are obtained by intersecting the predicted demand and supply. Anderson and Davison (2007) model demand and available generation capacity with outages and repair times and use the ratio of demand to capacity as an indicator of price spikes likelihood. Kwok and Sherris (2005) model demand and supply curves (bid stacks) submitted to NEMMCO and forecast prices based on the intersection of the two.

Furthermore, empirical work on electricity prices includes Lucia and Schwartz (2002) and Escribano et al. (2002) who use daily average prices. Kellerhals (2001) considers hourly spot prices data and calibrates stochastic volatility model of the forward rate. Longstaff and Wang (2004) conduct an empirical analysis of electricity forward prices using hourly spot and day-ahead forward prices. Other papers like Gibson and Schwartz (1990), Amin, Ng and Pirrong (1995), Kaminski (1997) and Eydeland and Geman (1998) focus on pricing energy contracts.

New Approach in Pricing Electricity Forwards The idea of this paper stems from a new actuarial approach recently developed by Elliott and van der Hoek (2003) for pricing real options. In real option pricing, the problem is that the underlying asset may not be tradeable. In the finance literature, there seem to be two solutions to this problem:

1. Assume that the non-tradeable underlying asset can be replicated (hedged) by a portfolio of tradeable assets. We can then synthetically trade the non-tradeable asset by using the traded assets which replicate it. This situation is described by saying there is a twin security which is tradeable. Under this assumption all the ideas from financial options carry over to real options

2. Develop some new principles for valuing options on non-tradeable assets, like the work of Smith and McCradle (1998), Henderson (2002) and Musiela and Zariphopoulou (2002), which has roots in the actuarial sciences. The approach is based on the concept of certainty equivalent. At wealth ω of the investor, the certainty equivalent $\overline{G}$ of an uncertain position G in one period's time is determined from

$$u(\omega + \overline{G}) = \mathbb{E}\left[u(\omega + G)\right] \tag{1}$$

where u is is a von-Neuman Morgenstern utility function and $\mathbb{E}$ is the expectation operator. There are many examples of utility functions and a popular one is the exponential utility. This is of the form

$$u(x) = -\exp(-\gamma \cdot x) \tag{2}$$

for some $\gamma > 0$. The number γ measures the level of risk aversion of the investor.

The new approach developed by Elliott and van der Hoek (2003), Smith and McCradle (1998), Henderson (2002) and Musiela and Zariphopoulou (2002) could be regarded as a generalization of two extreme pricing methodologies: Risk-neutral pricing when options are on tradeable assets and the actuarial pricing when no hedging is possible. This overcomes the fact that the actuarial approach does not price derivatives on tradeable assets correctly and the risk neutral pricing cannot price unhedgeable claims. The idea is the following:

Let

$$V_0(x) = \max \mathbb{E}\left[u(X(1))\right] \tag{3}$$

where the maximum is taken over all portfolios in tradeable assets with the property that their initial value $X(0) = x$ and they have uncertain value $X(1)$ in one period's time. We also let

$$V_G(x) = \max \mathbb{E}\left[u(X(1) + G)\right] \tag{4}$$

where G is the value of the contingent claim at time 1 and the maximum is taken over all portfolios in tradeable assets with the property that their initial value $X(0) = x$. We then define $\nu^b(G)$, the bid price of G, as the solution of

$$V_G(x - \nu) = V_0(x) \tag{5}$$

In words, the equation above states that in the presence of optimal investment in tradeable assets at a given level of wealth x we are indifferent between ν now and G in one year's time.

The expression for $\nu^b(G)$ can be computed explicitly in the case of exponential and quadratic utilities, and numerically for a wide range of other utility functions. In a similar way we can define $\nu^a(G)$, the asking price for G, and it can be shown that $\nu^a(G) = -\nu^b(-G)$. Good approximations are available for the computation of ν^a and ν^b (see Elliott and van der Hoek (2004)).

In the electricity market, consumers are price takers. Most forward contracts are written on some average of electricity prices, taken over some portion of each day for some number of days of the year, typically a multiple of a quarter of a year. The average price is an ask price quoted by intermediaries such as brokers or directly by traders. The dynamics for the average allows us to identify the risk aversion of the market players. The forward contract on a quantity Q is a contract to receive "floating" payments for fixed payment $F(Q)$ per unit. $F(Q)$ is determined so that the asking price for entering this contract is zero.

3 Forward Contracts

3.1 Definition of Electricity Forwards

An electricity forward (or swap) is an agreement between two parties to exchange a fixed set of payments against a floating set of payments over a period of time. The fixed set of payments corresponds to the

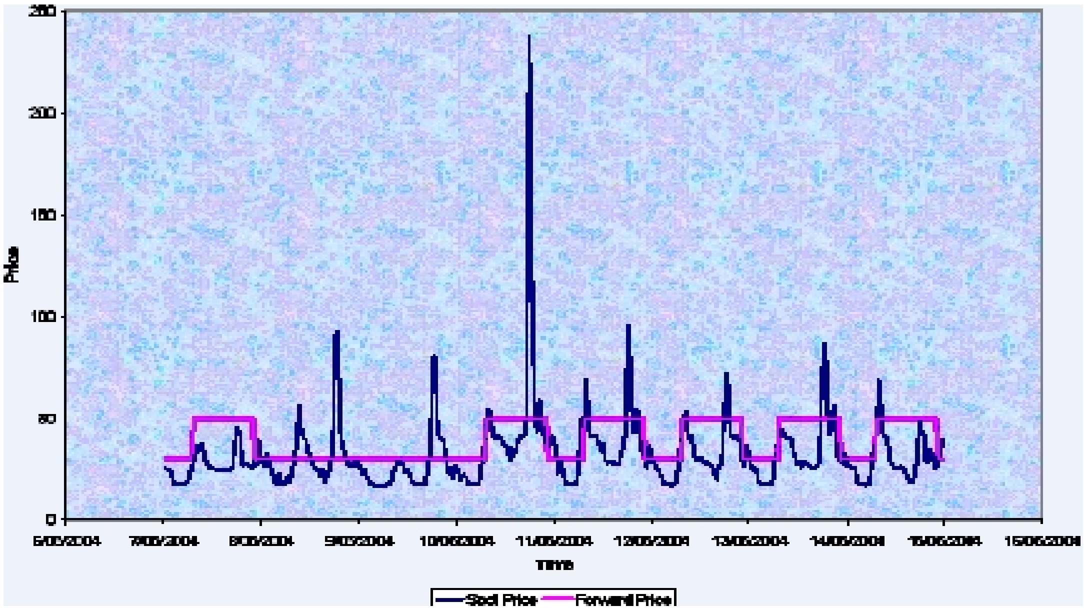

Figure 2: Peak/off-peak forward electircity contract

settlement of a specific notional quantity of electricity (in MWh) at a fixed price. The floating set of payments corresponds to the settlement of the same notional quantity of electricity at a floating price. The fixed price of the swap is agreed between the parties at the inception of the contract. The floating price is typically the pool price determined by the NEM at the specified node in the contract, for example, NSW pool price. So, for every half-hour of the swap contract period, the swap buyer pays the fixed price (also referred to as the strike price) multiplied by the notional quantity of the swap (in MWh) for that period. In return, they receive the spot price for that half-hour multiplied by the notional quantity. This is illustrated in (Fig.2) which plots NSW pool prices for an 8 days period from May 7^{th}, 2004 to May 15^{th}, 2006. The forward price is partitioned by peak and off-peak periods.

As a net position, the swap buyer receives the difference between the spot price and the fixed price for every half-hour in the swap period multiplied by the notional quantity. This amount may be positive or negative depending on the level of the spot price relative to the strike price. Settlement of the half-hourly difference payments occur on a **weekly basis** defined by NEMMCO calendar (from Saturday to Saturday). Since the holder of a long forward contract pays the difference between the spot price and the forward contract price, the effectiveness of the forward contract can be seen when the spot price is above the strike price. If the spot price is less than the purchased contract price for electricity then the buyer is paying more per hour for their energy needs, effectively making small financial losses every half hour over the total period of the contract, accumulating larger net losses. This situation is referred to in the market jargon by "**bleeding to death**".

Forward contracts are priced in discrete 30 minute periods, and may be engineered to cover a variety of periods during the full period of the contract. For example they can cover all 30 minute periods over the contract period (flat contracts), cover only the 30 minute periods between 07:00 and 22:00 on weekdays (peak contracts) or can cover only specific periods required over the period of the contract (flexible contracts). The following categories are the most traded in the market:

1. Flat swaps: where the underlying volume is constant determined as the inception of the contract

2. Profiled swaps (also referred to as sculpted) where the underlying volume is variable but determined at the inception of the contract

3. Whole of meter swaps: is a swap linked to certain customer consumption, so the underlying volume is not known by certainty.

Within each of the above category, contracts can be one of the following types:

1. Peak

TERM	Peak Fwd Price	Flat Fwd Price	Off Peak Fwd Price
April 2007	50.500	44.250	31.250
May 2007	59.875	44.725	34.090
Q2 2007 (AMJ)	63.750	45.200	31.250
Q3 2007 (JAS)	61.750	43.750	29.714
Q4 2007 (OND)	60.000	40.750	25.830
Q1 2008 (JFM)	93.650	54.250	24.688
Fin Year 07/08	65.450	43.661	26.504
Cal 08	62.900	42.200	26.500
Fin Year 08/09	59.600	41.325	27.218
Cal 09	57.750	40.825	26.160
Fin Year 09/10	58.850	40.500	25.698
Cal 10	59.000	40.600	26.458
Fin Year 10/11	58.508	40.400	26.458
Cal 11	59.000	40.450	25.649

Figure 3: AFMA forward curve on March 1st, 2007

2. Off-peak
3. Flat (48 half-hours)

A peak swap for instance yields difference payments only for peak hours (7am to 10pm). In the NEM, electricity swaps are liquid forward contracts. They are widely available through brokers and also in the OTC market. They are defined on future periods, typically, the following month, quarters and financial and calendar years. The Australian Financial Mathematics Association surveys all market participants about forward prices and ask them to provide bid & offer prices for the following periods on a daily basis:

- Next two (2) full calendar months
- Four (4) financial quarters
- Next four (4) full financial years
- Next four (4) full calendar years

Figure (3) provides a snapshot of the AFMA forward curve taken on March 1^{st}, 2007. The Peak Fwd Price column provides forward prices for peak periods in different terms in the future. For example, \$50.50 for a forward contract covering peak periods in April 2007. The underlying volume for the swap contracts is 10 MW. Peak contracts are defined as per AFMA Market Conventions, which are available from www.afma.com.au.

3.2 Relationship between Forward Price and Spot Price Average

A swap contract has no premium associated to it. The forward price is set in such a way that it costs nothing to enter the swap. Let S_t denote the spot price of electricity at the end of the half-hour t and F the forward price set at the beginning of the swap contract. Let us assume that the underlying volume of the flat swap is $2L$ MW[1]. Also, let M_S be the number of half-hours contracted in the swap period. The contract can be peak, off-peak or flat. The swap buyer will receive the difference payments of the amount

$$\sum_{t=1}^{M_S} (S_t - F) L = L \left(-F M_S + \sum_{t=1}^{M_S} S_t \right) \tag{6}$$

[1] A flat swap contract with undelying volume of $2L$ MW corresponds to L MWh energy per half-hour.

It is worth noting that at the time when the swap is bought, which is about the time when F is determined, the future values of spot prices, S_t are not known. If these values were known, then, given the zero cost of the swap, the forward price F could be chosen such that

$$F = \frac{1}{M_S} \sum_{t=1}^{M_S} S_t$$

However, since future prices S_t, for $t \in [0, M_S]$ are not known at time 0, the question arises regarding the fair value of F given expectations of future prices S_t. The no-arbitrage or cost of carry approach assumes that investors can synthesize a forward contract by taking a long position in the underlying asset and holding it until the contract expiration date. If the forward price does not equal the price of the replicating portfolio, then arbitrage profits are possible. Thus, the forward price is linked directly to the current spot price by the following relationship

$$F = \frac{1}{M_S} \sum_{t=1}^{M_S} \mathbb{E}^{\mathbf{Q}} [S_t]$$

where $\mathbb{E}^{\mathbf{Q}} [\bullet]$ is the expectations under an equivalent risk-neutral measure $\mathbf{Q}$.

It is important to note that the no-arbitrage argument underlying this model relies on the ability of an arbitrageur to take a position in the underlying asset and hold it until the contract expiration date. Since electricity is essentially not storable, however, the cost-of-carry model cannot be applied directly to electricity forward prices. Therefore, in this research project we do not adopt such approach.

If we suppose that the swap spans N weeks: $M_S = \sum_{w=1}^{N} M_w$ where M_w denotes the number of hours contracted for week w, then Equation (6) can be rewritten as:

$$\begin{aligned}
\sum_{t=1}^{M_S} (S_t - F) L &= L \left(-F M_S + \sum_{t=1}^{M_S} S_t \right) \\
&= L \left(\sum_{w=1}^{N} \left[-F M_w + \sum_{t=1}^{M_w} S_t \right] \right) \\
&= L \left(\sum_{w=1}^{N} M_w (P_w - F) \right)
\end{aligned}$$

where P_w is the average of the pool prices over week w. Since the payoff of the swap contract is written in terms of the pool price average, we shall focus on the modelling of the weekly pool price average rather than the half-hourly prices.

To illustrate the relationship between spot and forward prices, figure (4) plots two time series of forward prices and spot price averages. The blue line is the average of forward prices available at each point in time. For example, on 14 July 2003, the average forward price will be based on the forward prices for the periods: calendar year 2004 and 2005 and financial year 03/04. The pink line in this figure is the moving average of spot prices looking one year back. It is obvious from the graph that throughout the period 1999 to 2004, there is a strong correlation between the two variables. The orange bars in the graph represent a 90-day rolling correlation between the two time series. It is clear that the correlation approaches 1 and -1 and oscillates between the two values. If at each point in time, the average of one year historical spot prices can be considered as a proxy for the conditional expectations of future spot prices, then it is clear from the graph in figure (4) that forward prices can be modelled as a function of expected spot prices.

A survey of literature (see Johnson and Barz (2005), Eydeland and Geman (2005) and Kosecki (2005)) suggests that the relationship between the forward contract prices and the spot price involves four factors:

1. Timing of buying the contract
2. Contract underlying volume
3. Generator expectation of the market demand and its units capacity
4. Current spot price and expectation of future spot prices.

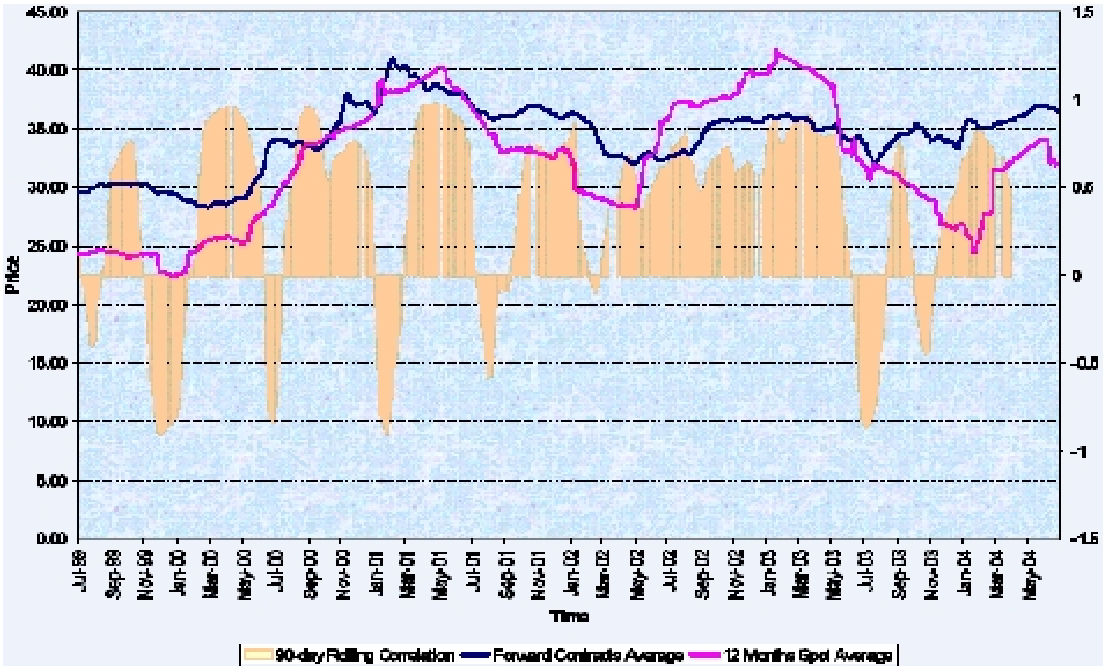

Figure 4: Correlation between forward price and spot price average

4 Stylised Facts of Electricity Prices

It is well known in the same literature and from empirical investigation that electricity spot prices exhibit cycles, seasonality and autocorrelation. This data structure can be captured and used in the process of forecasting future spot prices. We also know that occasional price spikes may occur due to unusual high load, network transmission constraints and unscheduled outages of generation. This is because electricity is economically not storable, so sudden high demand should instantaneously be matched by an increased supply. This causes prices to increase if there is not enough capacity in the region to respond to this need. Unlike other types of commodities, high demand shocks cannot be smoothed by inventory. These extreme values and their probability of occurrence can also be captured from historical data and used in the modelling of future prices.

Furthermore, prices also exhibit different behaviours depending on whether they fall in peak or off-peak periods. This reflects the economic law of supply and demand. In peak hours, demand is high so prices are more likely to rise. In off-peak hours, demand is low so prices have tendency to drop. For this reason, it is important to consider two time partitions: peak and off-peak, in the modelling of the dynamics of the spot price averages as a regime switching time series. The peak period is defined from 7:00 am to 10:00 p.m. during the weekdays, and the off-peak period is from 10:00 p.m. to 7:00 am for weekdays and all weekends and public holidays. This corresponds to the definition of the NEM for the NSW market. Figure (5) represents half-hourly NSW spot prices for the period between 2001 and 2007. It is clear that the risk included in the recurrent spikes is significant.

Figure (6) shows summary statistics of half-hourly spot electricity prices for NSW reported by the NEM. Prices are quoted in terms of \$/MWh. The sample consists of daily observations of the 48 half-hourly spot prices during the March 1^{st}, 2001 to February 28^{th}, 2007 (inclusive) period. Two consecutive half-hours are aggregated in the one hour bucket. For example, Hour 7 comprises the 7am-7:30am and 7:30am-8:00am half-hours periods. The risk (measured by the standard deviation) varies throughout the day. It peaks in the morning (breakfast time) and most of the afternoon. The coefficient of variation (ratio of standard deviation to the mean) illustrating this pattern is plotted in (Fig.7).

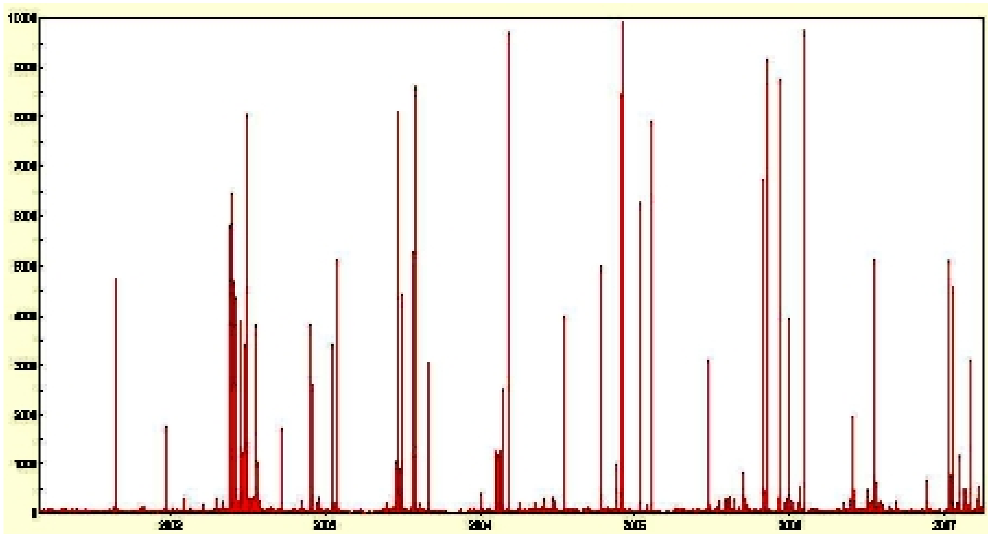

Figure 5: NSW pool prices from March 1^{st}, 2001 to February 28^{th}, 2007

Hour	Mean	Std. Deviation	Minimum	Median	Maximum
1	23.44	10.10	9.76	21.40	197.20
2	20.61	10.00	4.99	18.09	260.18
3	17.33	6.85	1.47	16.00	99.00
4	15.74	5.43	0.00	14.99	90.21
5	17.27	6.24	0.00	16.20	107.80
6	22.11	31.94	0.00	19.27	1411.63
7	30.09	117.30	4.42	22.77	4036.27
8	29.19	15.34	10.07	25.30	441.28
9	31.47	22.38	11.60	27.67	864.60
10	31.60	20.44	11.79	27.70	610.30
11	32.00	30.17	11.60	27.25	932.48
12	34.96	95.38	10.36	26.74	5113.60
13	41.50	165.40	9.72	26.29	4965.30
14	50.54	297.39	7.09	25.71	9612.61
15	53.08	352.23	8.16	25.00	9909.03
16	53.28	349.59	9.44	25.00	9488.59
17	40.82	198.04	9.23	26.05	9083.62
18	89.51	457.12	10.43	30.85	8622.63
19	41.83	130.29	10.44	29.22	5264.02
20	30.12	40.34	9.35	26.00	2568.73
21	26.32	15.63	9.78	23.49	705.88
22	25.45	15.98	10.29	22.91	807.99
23	28.14	40.08	11.18	25.17	2758.89
24	25.36	11.00	10.24	23.58	210.76
Peak	41.09	173.16	4.42	26.05	9909.03
Offpeak	21.72	15.96	0.00	19.27	2758.89
Overall	33.82	166.28	0.00	25.09	9909.03

Figure 6: Summary statistics of half-hourly NSW spot prices

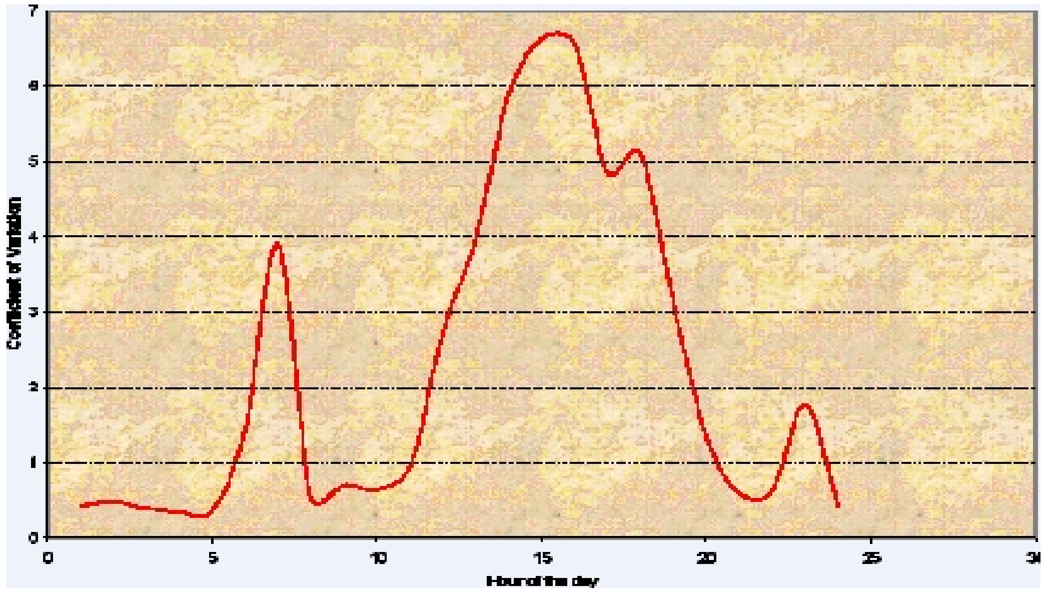

Figure 7: Coefficient of variation in the half-hourly NSW spot prices

4.1 A Digression on Electricity Returns - Compass Rose

The "Compass Rose" pattern was identified in the literature by Crack and Ledoit (1996). It appears in scatter diagrams of percentage stock returns against their lagged values. In this diagram, rays that emanate from the center can be discerned, and resemble a compass rose (see Fig.8).

The formation of these rays is attributed to price clustering, discreteness and tick size (see Vorlow (2004)). These factors are considered to be an important part of the market microstructure literature with serious implications on risk evaluation, the optimal design of securities and market efficiency.

Most papers in the literature consider daily stock returns. Here we investigate whether we can observe the compass rose in half-hourly electricity prices. This exercise is somehow subject to a subjective judgement, and may not be useful in forecasting. However, it would be worthwhile to see if any obvious patterns occur in electricity prices and try to explain them.

Firstly, we form the returns series (R_t) where

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

and P_t is the NSW spot electricity price at time t. A history going from March 1^{st}, 1996 to May 20^{th}, 2007 is considered and the scatter plot of R_t versus R_{t-1} is displayed in Figure (9).

It can be seen from this graph that there are four apparent rays going from the center and also an exponential curve going through the centre. In order to explain the existence of this curve, half-hourly data was partitioned into seasons, months and peak/off-peak periods of the day. Figure (10) shows the scatter plot where all dots representing returns within peak periods are blue and off-peak returns are orange. The exponential curve persists where dots from both partitions are present. This suggests that this negative serial correlation does not depend on the time of the day. A similar graph for partition by season shows no evidence that this curve is attributed to returns during a particular time of the year.

The pairs in the top-left quadrant indicate negative returns followed by positive returns in the next period (half hour). Having them clustered in an exponential fashion, suggests that when prices fall below a certain level there is an immediate systematic response from some of the generators in NSW (major players like Macquarie Generator) who would withdraw capacity, say 100MW from the market to decrease supply so that prices then rise as a result of the law of supply and demand.

For the pairs in bottom-right quadrant, this is the case of positive returns followed by negative ones. So, as soon as prices rise to a certain level, there is an immediate systematic capacity offered by the gen-

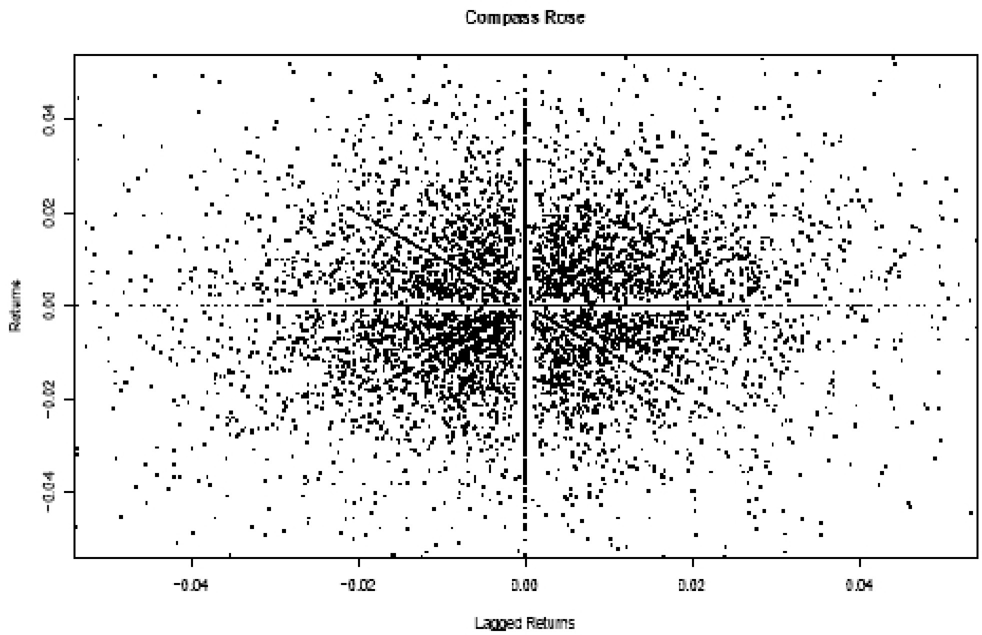

Figure 8: Stock returns compass rose

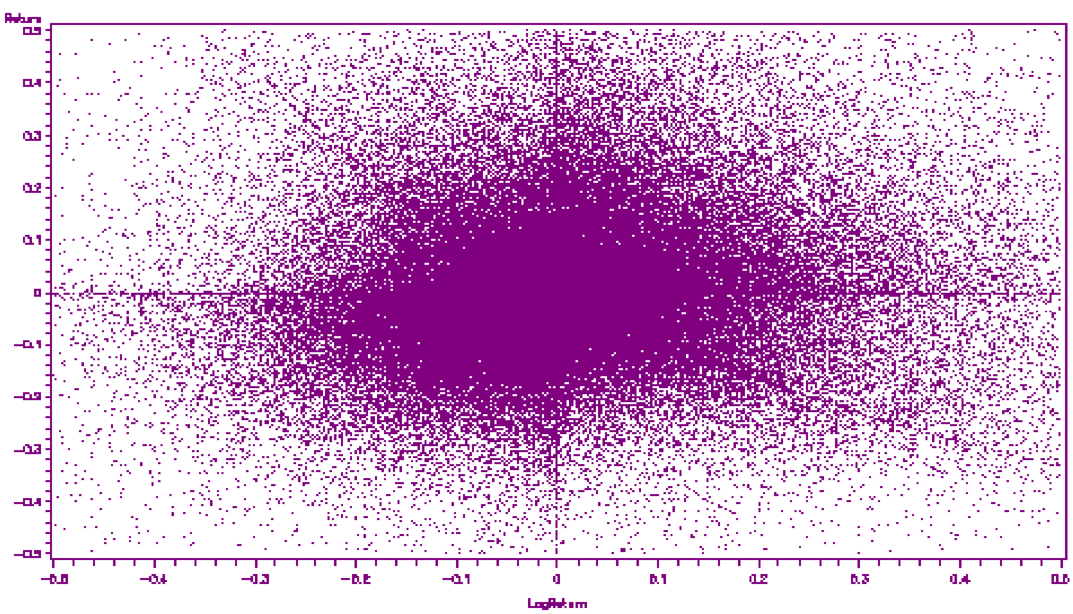

Figure 9: NSW electricity returns - compass rose for returns within [-50%, 50%]

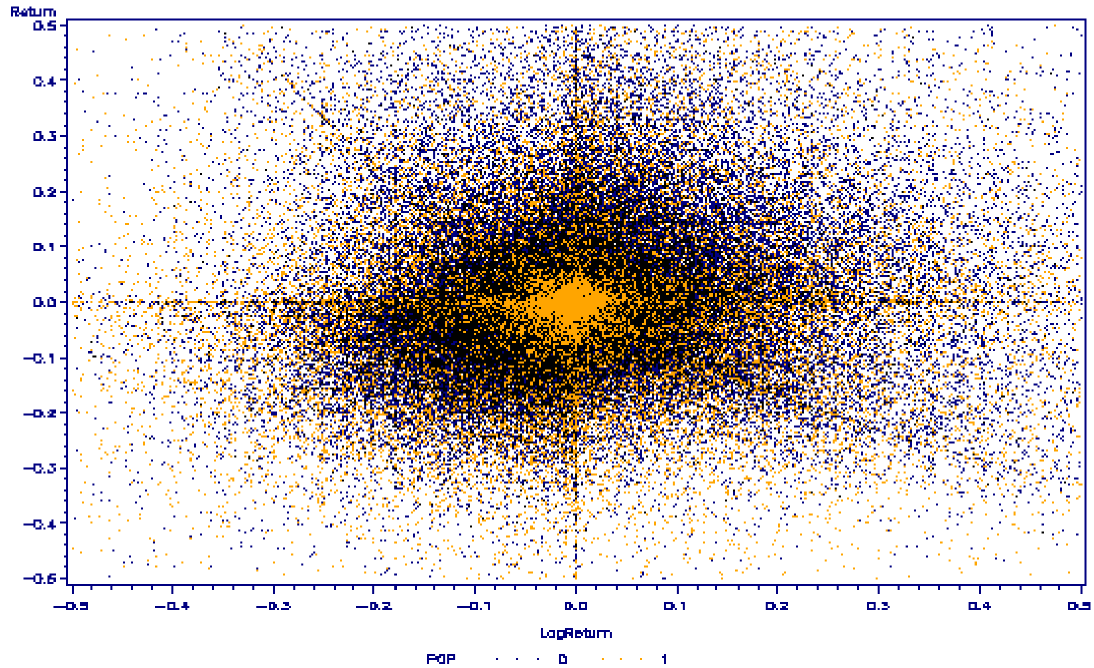

Figure 10: NSW electricity returns by peak and off-peak periods - compass rose

erator (100MW) to take advantage of high prices. This action of over supply then pushes prices down as a consequence.

The curve is not linear because the effect of withdrawing, or supplying, of the systematic capacity (100MW) is not linear on prices. So this may suggest inefficiency in the electricity market, and an arbitrage opportunity for some generators. however, when zooming into returns within 1%, the standard compass rose appears as per Figure (11).

5 Dynamics of Spot Price Average

The data analysis is based on NSW electricity historical pool prices which are available for each half-hour since March 1^{st}, 1996. The dataset needs to contain a sufficient number of observations to constitute a representative sample that appropriately reflects the statistical properties over time and that are sufficiently recent to still be relevant. Given the nature of the NEM which is relatively new, there has been undergoing several structural changes. In the past, there was introduction of the regulatory ETEF (Electricity Tariff Equalisation Fund)[2] and changes in the VOLL[3] (Value of lost load) from $5,000 to $10,000. There also has been an addition of new interconnectors between the Australian states. For this study, we consider half-hourly NSW pool prices from March 1^{st}, 2001 to February 28^{th}, 2007 inclusive. This corresponds to $105,168$ data points. When this data is partitioned, it is then comprised of $45,360$ and $59,808$ peak and off-peak data points respectively. When taking the weekly averages, this results in 313 data points for both partitions since there is the same number of weeks during the period considered, regardless of any partition.

[2] This is a government regulated fund that all market participants contribute to. Essentially, it is intended to manage the risk faced by a retailer who is supplying power to customers at a regulated tariff, while purchasing that power in the NEM.

[3] VOLL is a cap on Electricity spot prices imposed by the NEM. At the moment, there are expectations of increasing the VOLL to $30,000

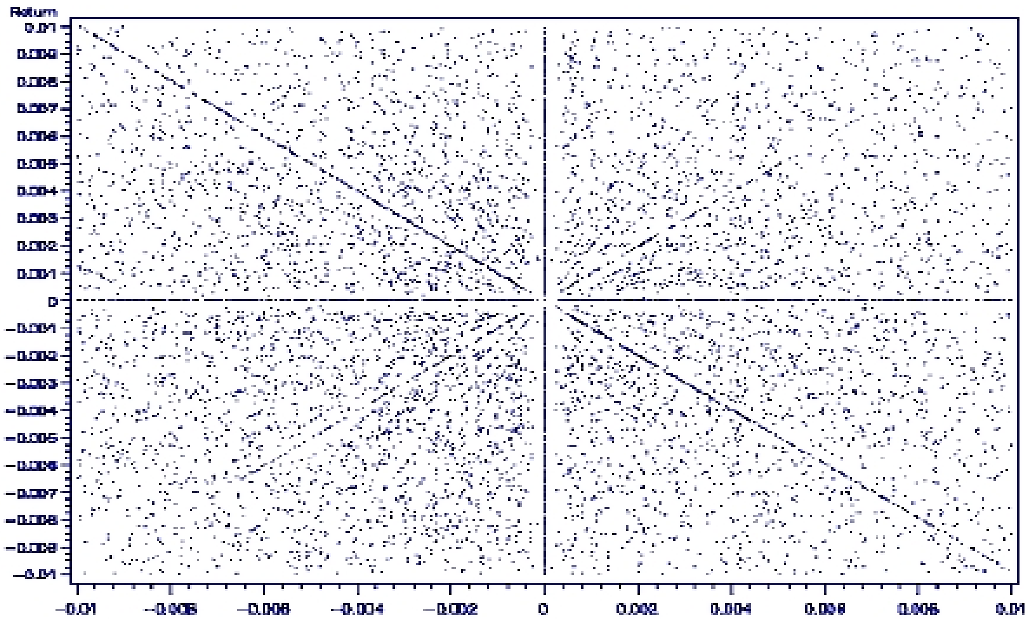

Figure 11: NSW electricity returns - compass rose, zoom into returns within [-1%, 1%]

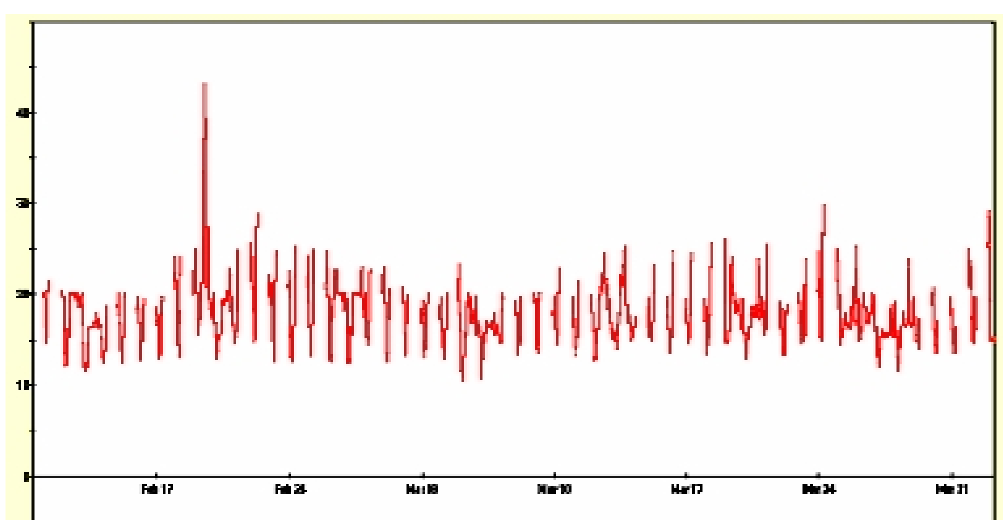

Figure 12: Off-peak partitioned NSW pool prices - 50 days period

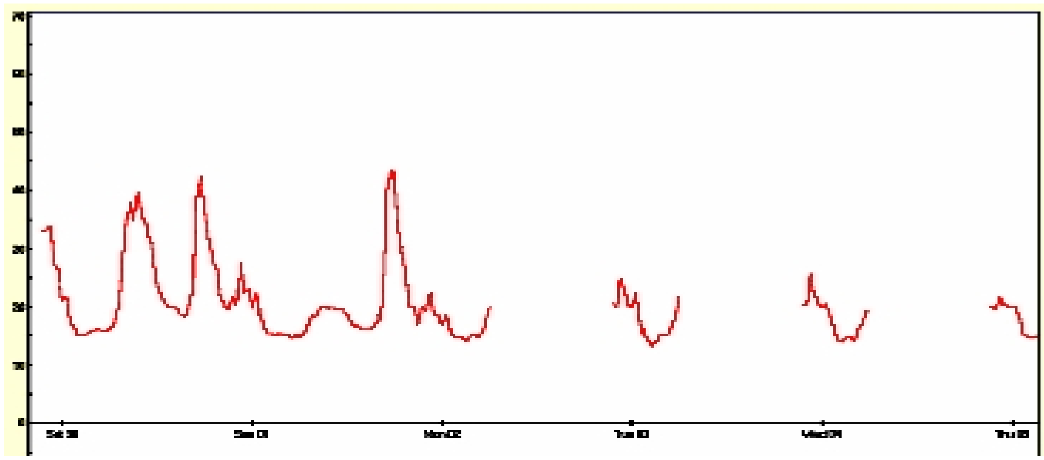

Figure 13: Off-peak partitioned NSW pool prices - 1 week period

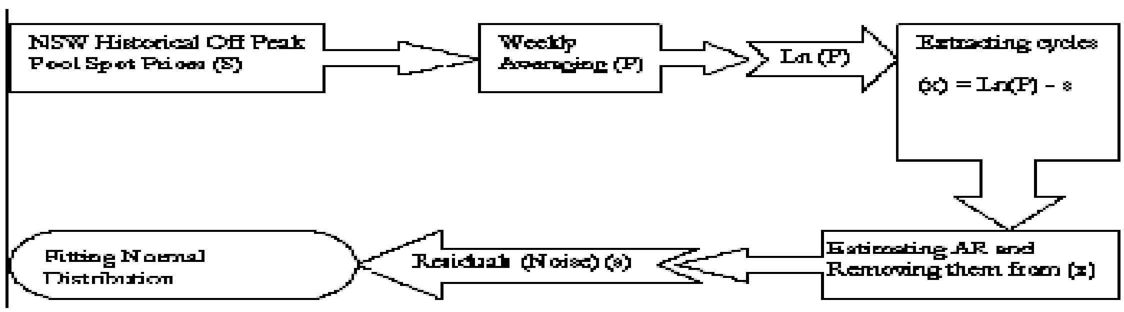

Figure 14: Steps in modelling weekly average of spot prices

5.1 Off-peak Electricity Price Average Dynamics

Geman and Roncoroni (2002) argue that it is not appropriate to model electricity spot prices using time series. Indeed, the strong serial correlation invalidates the usual assumption of independent identically distributed residuals. Also, the normality assumption for residuals does not hold because of the spikes. However, when taking the weekly averages, the spikes and the serial correlation are smoothed and the data can be modeled using time series analysis. Also, taking the weekly averages eliminates the daily and weekly cycles.

So, we proceed with a time series analysis of off-peak historical spot prices with the same modelling applied to peak data. Initially, we take the moving averages with a one week window and one week step of the historical data, then apply this to a logarithmic function. This has the effect of dampening the extreme values observed historically and makes the data more able to be analysed by other time series methods. Following this, yearly cyclic effect present in the data is estimated. Once this seasonality is estimated, it is removed so that the series remaining is a representation of the logarithmic of the historic underlying that has been de-seasonalised.

A key feature of the modelling of electricity spot prices is the maintenance of observed serial correlation structure in the series. Estimation of this structure can be done using autoregressive models which are used to estimate and remove the temporal correlations from the time series. After this process, the final residuals of the analysis, after the removal of seasonality and correlation should represent values that have low correlation and can be estimated from a normal distribution. These steps are summarised in Figure (14).

Figure (15) plots the weekly averages of NSW off-peak spot prices for the period March 1^{st}, 2001 to February 28^{th}, 2007. It can be seen from this figure that the spikes are smoothed out when taking averages. Figure (16) shows the autocorrelation structure of this time series. Each bar in the graph represents the value of the correlation between prices with a number of weeks lags. This figure shows a strong serial correlation

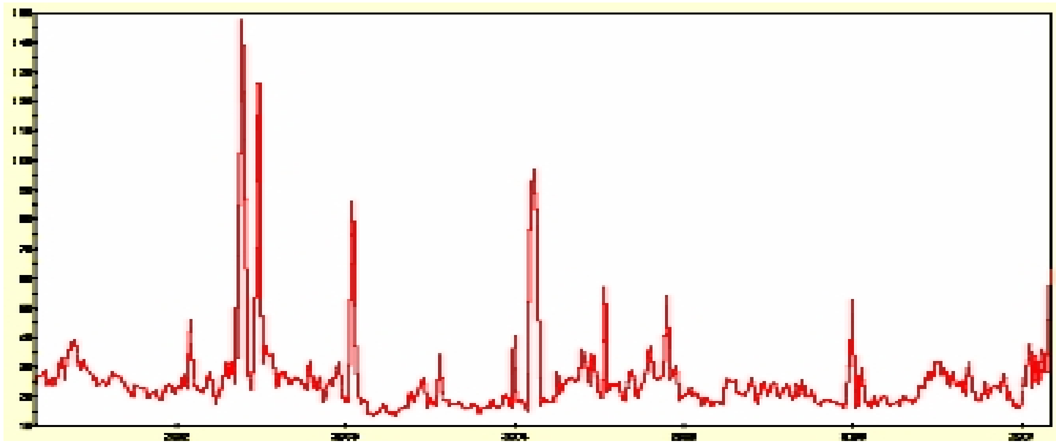

Figure 15: Weekly average of pool prices for off-peak periods

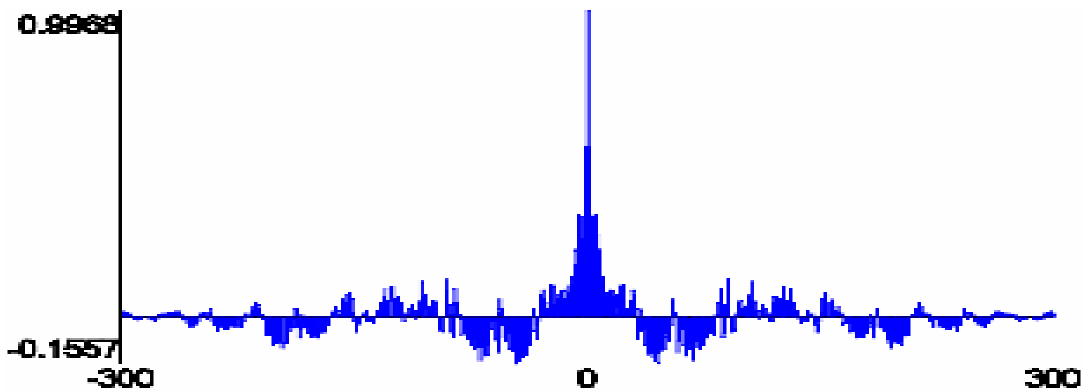

Figure 16: Autocorrelation in the off-peak price weekly averages

with a cyclical behaviour.

Figure (18) shows the yearly seasonal component of the prices. It is apparent from the plot that the weekly average prices drop below the yearly average in shoulder seasons (Autumn and Spring) and rise above the average for peak seasons. It is also clear that the winter peak is much higher than the summer one. Since prices are induced by demand, this suggests that in NSW, demand is more sensitive to cold than hot weather.

The logarithm of weekly average price P contains a yearly seasonal component denoted s. After removing this seasonal component. i.e. $x_t = ln(P_t) - s_t$, we propose to model x as an AutoRegressive model $AR(p)$ where p, the number of lags is determined using the BIC criterion.

After fitting a linear model to (x_t), we obtain an $AR(3)$ model

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \phi_3 x_{t-3} + \sigma \varepsilon_t$$

with

$$\begin{aligned} \phi_1 &= 0.223063 \\ \phi_2 &= -0.215657 \\ \phi_3 &= -0.236357 \\ \sigma^2 &= 0.0526866 \end{aligned}$$

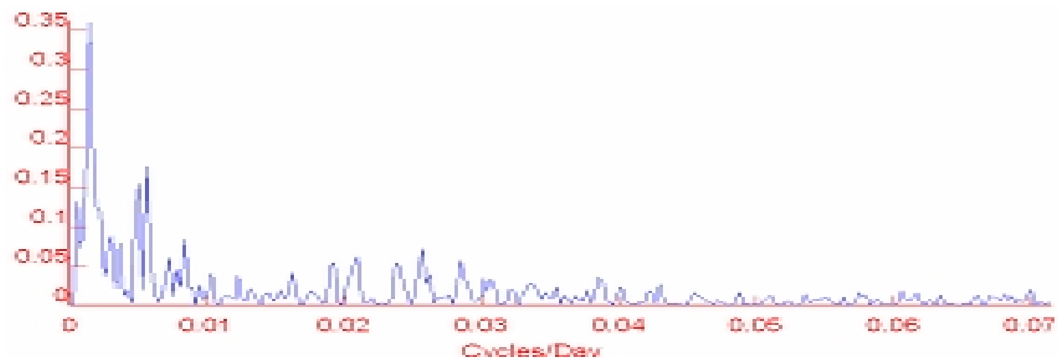

Figure 17: Fast fourrier transform of the off-peak price weekly averages

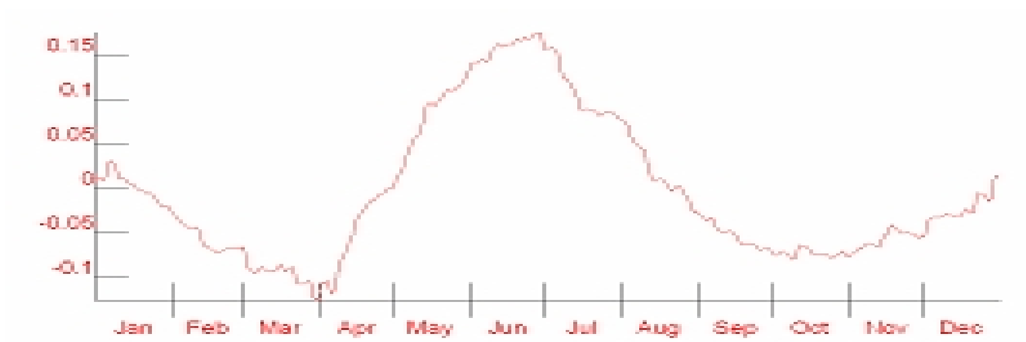

Figure 18: Seasonality in the pool price weekly averages

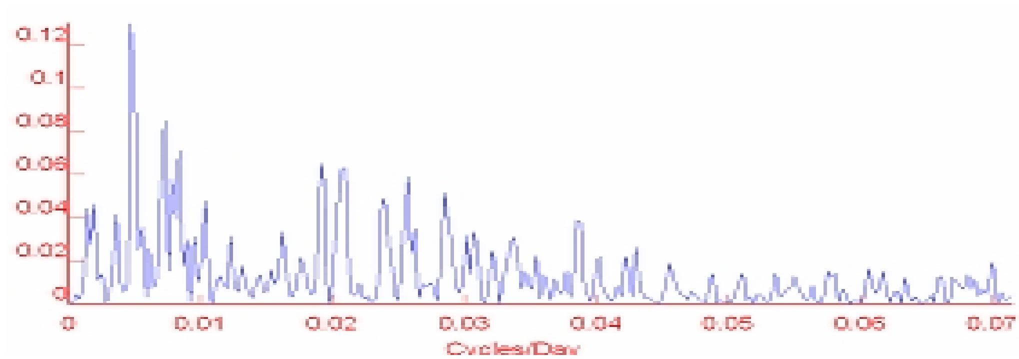

Figure 19: Fast fourrier transform of de-seasonalised price weekly averages

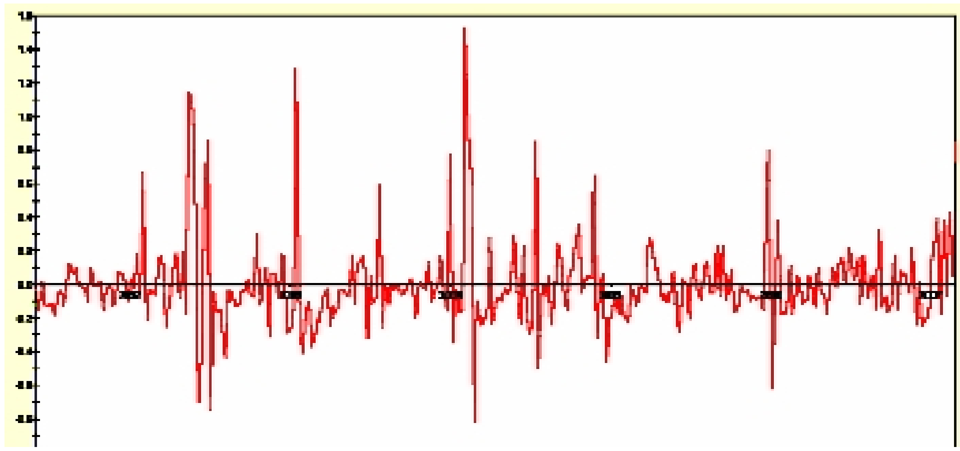

Figure 20: Plot of residuals

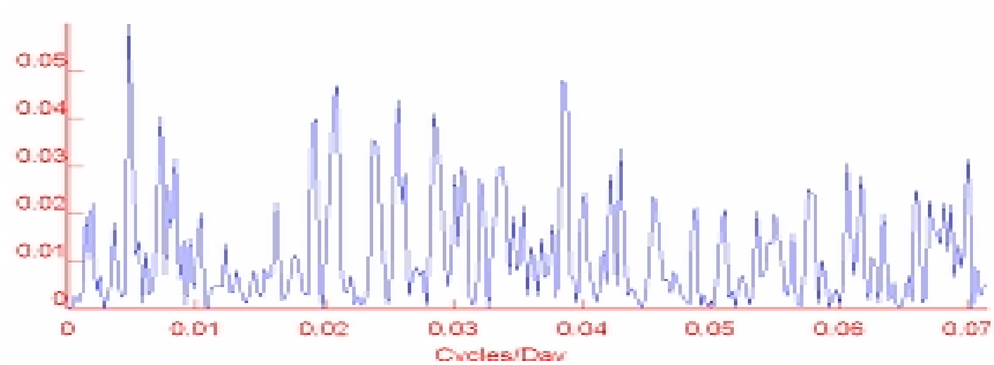

Figure 21: Fast fourrier transform of the residuals

The resulting BIC value is equal to -1.25419. Figure (22) shows the autocorrelation structure of the residual terms of the model. Compared to Figure (16), the autocorrelation has decreased from a maximum absolute value of 50% to 13% and the overall structure is scarce. This demonstrates that the filtering applied to the data has captured the autocorrelation. Figure (23) shows the frequency distribution of the residuals. A Jarque-Bera test for normality resulted in rejecting the normality of residuals at 5% confidence level (the p-value 9.72%).

5.2 Peak Electricity Price Average Dynamics

We apply the same methodology as the above to estimate the process governing the dynamics of the average peak prices from March 1^{st}, 2001 to February 28^{th}, 2007 inclusive. The fitted model is an $AR(4)$ with the parameters:

$$\begin{aligned}\phi_1 &= 0.0142917\\ \phi_2 &= -0.208384\\ \phi_3 &= -0.22826\\ \phi_4 &= -0.18266\\ \sigma^2 &= 0.214334\end{aligned}$$

The resulting BIC value is equal to -0.636761.

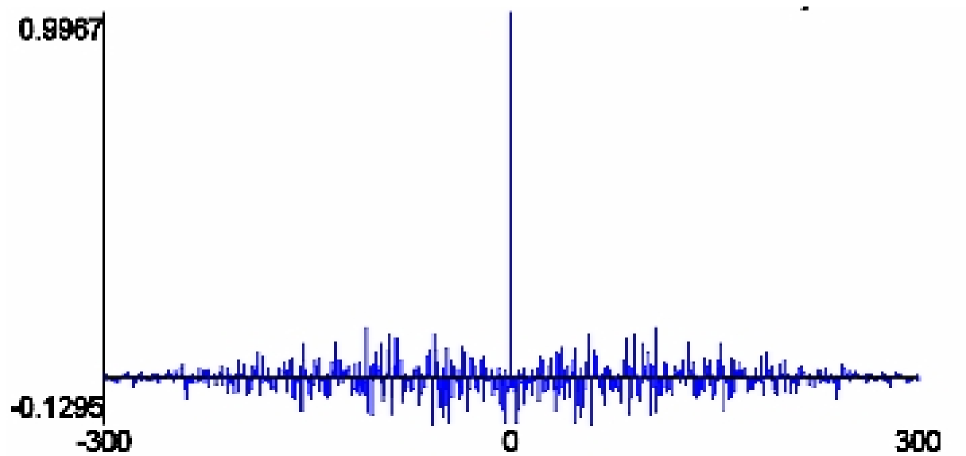

Figure 22: Autocorrelation structure of the residuals

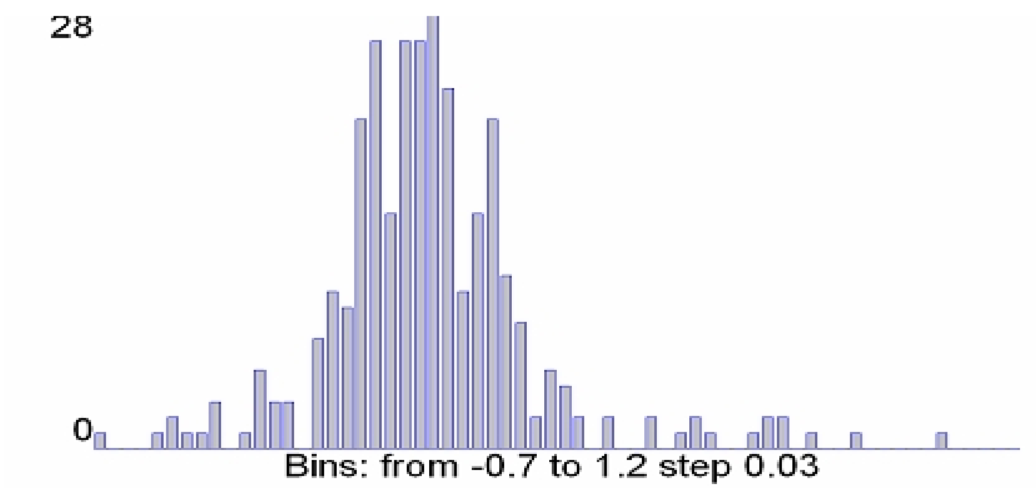

Figure 23: Frequency distribution of the residuals

6 Electricity Forwards Asking Prices

In this section we use real option theory for the pricing of electricity forwards. The pricing methodology developed here is independent of the model for the dynamics of underlying given in the previous section. In effect, the resulting forward price is a solution of a system of equations involving conditional expectations and variances of the average prices. It is at the stage of finding an explicit solution where the model for the dynamics of the underlying developed in the previous section is used.

6.1 An Approximative Valuation Formula for Contingent Claim Premium

We consider a one period model and let the generator have expected utility preferences over uncertain outcomes at the end of the period. If π is the price at the beginning of the period of an asset that has (random) payoff Z at the end of the period, then at an initial wealth $X(0)$, the π should solve the following equation

$$\mathbb{E}\left[u\left(X(0)\right)\right] = \mathbb{E}\left[u\left(X(1) + \pi R - Z\right)\right] \tag{7}$$

where R is the compounded interest and $X(1)$ is terminal wealth. We interpret this as follows: at wealth W we are different between W and giving up Z at the end of the period and receiving π at the beginning (which is πR at the end of the period).

If W is deterministic and $u(x) = -\exp(-\gamma x)$ then (7) implies

$$\pi = \frac{1}{\gamma R} \ln\left(\mathbb{E}\left[\exp\left(\gamma Z\right)\right]\right)$$

π is referred to as the certainty equivalent of the contingent claim Z. In the special case when $Z \sim N(\mu, \sigma^2)$, then

$$\pi = \frac{1}{R}\left[\mu + \frac{1}{2}\gamma\sigma^2\right]$$

In the presence of optimal portfolios, a more general pricing approach introduced by Elliot and van der Hoek (2003) consists of defining:

$$V_G(x) = \max \mathbb{E}\left[u(X(1) + G)\right] \tag{8}$$

where G and $X(1)$ are respectively the value of the contingent claim and wealth at time 1 and the maximum is taken over all portfolios in tradeable assets with the property that their initial value $X(0) = x$. Then $\pi^b(G)$, the bid price of G, is defined as the solution of

$$V_G(x - \pi) = V_0(x) \tag{9}$$

as explained in section (2). Using Taylor's expansion, it can be shown (see Elliot and van der Hoek (2003)) that equation (9) can be solved for π using the the following approximation formula

$$\boldsymbol{\pi} \approx \frac{1}{R}\left[\mathbb{E}\left[G\right] + \frac{1}{2}\frac{u''(W)}{u'(W)}\mathbf{var}\left[G\right]\right] \tag{10}$$

where u, the utility function is general. We shall refer to the above equation as the valuation formula, and will be used in later sections as a basis for pricing electricity forward contracts. Electricity prices as regarded as asking prices (asked by the generators/retailers) and the electricity users/consumers are price takers.

Skantze and Ilic (2000) model the forward price as a function of the spot price, the variance of the spot price, and a random disturbance (z_F),

$$F(t,T) = \Phi\left(E_t\left[S_T\right], var_t\left[S_T\right], z_F\right)$$

where the exact form of Φ is likely to vary from market to market. They solved for $F(t,T)$ using stochastic dynamic programming.

lambda	min	max	values < 1	values < 0
0.4	0.14	7.43	11%	0
0.473	0.0023	5.21	40%	0
0.5	-0.003	4.62	51%	1
0.6	-0.118	3.11	87%	6

Table 1: Optimal value for the risk premium parameter lambda

6.2 Application to Electricity Markets

Let P_t denote the average of electricity pool price over week t. Let $\mathbb{E}_t[X]$ and $var_t[X]$ respectively denote the expectation and the variance of the random variable X conditional on the natural filtration $\mathcal{F}_t$ of the price P_t based on the information up to time t, i.e. $\mathcal{F}_t = \bigcup_{s\leq t}\sigma(P_s)$. We also denote by $cov_t(X,Y)$ the covariance of the two random variables X and Y conditional on $\mathcal{F}_t$. Using the notation in section (6.1), we postulate

$$\pi = P_{t-1} \text{ and } Z = \lambda P_t \tag{11}$$

where $0 < \lambda < 1$ is a constant. By analogy from commodity pricing, equation (11) translates the fact that the price of electricity today is the certainty equivalent of the next period's price, adjusted by a constant λ. For this, λ can be regarded as the electricity market price of risk.

Applying the valuation formula **(10)**, and using exponential utility function, we obtain

$$P_{t-1} = \frac{1}{R}\left[\mathbb{E}_{t-1}[\lambda P_t] + \frac{\gamma}{2}var_{t-1}[\lambda P_t]\right]$$

The risk aversion parameter γ_{t-1} can be inferred from the above as follows

$$\gamma_{t-1} = \frac{-\mathbb{E}_{t-1}[\lambda P_t] + RP_{t-1}}{\frac{\lambda^2}{2}var_{t-1}[P_t]} \tag{12}$$

where $R = e^{r\Delta t}$, r is the risk-free rate of interest and $\Delta t = 1/52$ (1 week).

Section (5) models $(P_t)_{t\geq 0}$ as a lognormal process (weekly average prices are positive). So, let $P_t = e^{p_t}$ where $p_t \sim \mathcal{N}(\mu_{t-1}, \sigma^2)$ and $\mu_{t-1} = s_t + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \phi_3 x_{t-3}$, $\sigma > 0$. We have

$$\begin{aligned} \mathbb{E}_{t-1}[\lambda P_t] &= \lambda e^{\mu + \frac{1}{2}\sigma^2}, \text{ and} \\ var_{t-1}[\lambda P_t] &= \lambda^2 e^{2\mu}\left(e^{2\sigma^2} - e^{\sigma^2}\right) \end{aligned}$$

So:

$$\gamma_{t-1} = \frac{RP_{t-1} - \lambda e^{\mu+\sigma^2/2}}{\frac{\lambda^2}{2}e^{2\mu}\left(e^{2\sigma^2} - e^{\sigma^2}\right)}$$

Figure (24) plots the values of the process (γ_t), representing the market risk aversion parameter over time,

given a value of $\lambda = 0.5$, for the period from March 1^{st}, 2001 to February 28^{th}, 2007, amounting to 312 observations (weeks).

The values of lambda are chosen in such a way that the risk aversion parameter γ is positive and not too large. For different values of λ we compute the minimum, maximum, percentage of the number of the values of γ less than 1 and the number of values less than zero. Table (1) summarises the results

This analysis suggests that the value of $\lambda = 0.473$ is optimal for there are no negative values of γ and the percentage of values less than 1 is the highest.

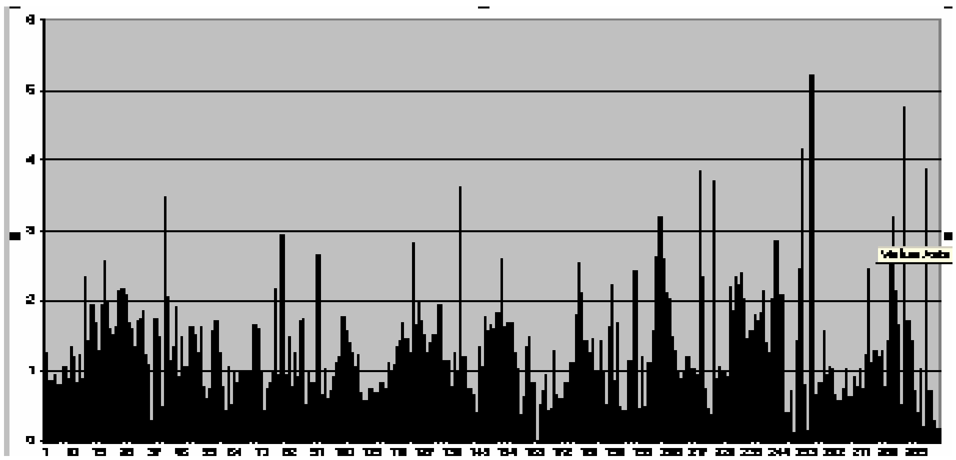

Figure 24: Risk aversion parameter γ values of time for $\lambda = 0.5$

6.3 A Recursion Formula for Electricity Forward Prices

In this section, we shall consider a general class of swaps where the amount $\overline{L}_w$ (MWh) of energy contracted for week w can vary from week to another during the life of the forward contract. The only restriction is that within week w, the load rate L_w (MW) is constant for each half-hour contracted in this week. These contracts can be peak, off-peak or flat half-hours. As per the NEM convention, we assume that contracts are settled each week. The settlement amount is

$$\sum_{i=1}^{M_w} (S_i - F) \frac{L_w}{2} = M_w \frac{L_w}{2} [P_w - F]$$

where M_w is the number of half-hours contracted in week w, S_i (\$/MWh) is the pool spot price for the half-hour i, P_w is the average price for week w and F is the strike price of the contract (forward price per MWh). M_w could depend on whether the underlying contract is flat, peak or off-peak. For example, if the underlying contract is a NSW off-peak then for the week starting on 02/02/2004, the number of half-hours contracted is[4] $M_w = 191$.

Let us write

$$\overline{L}_w = M_w \frac{L_w}{2}$$

the amount of MWh used in week w. Let $F_t(n)$ denote the forward price at time t corresponding to payments

$$\overline{L}_w [P_w - F]$$

at $w = t+1$, $t+2$, ..., $t+n$.

Let V_t^n be the present value of this contract (asking price), then $F_t(n)$ is the value of F such that $V_t^n = 0$.

As described in figure (25), V_t^n is the ask price of

$$\overline{L}_{t+1}(P_{t+1} - F) + V_{t+1}^{n-1} = Z_{t+1} \tag{13}$$

where $\overline{L}_{t+1}(P_{t+1} - F)$ is the settlement amount for next week and V_{t+1}^{n-1} is the present value of future settlements at time $t+1$ with $n-1$ remaining periods. This method of valuation, similar to option pricing, provides consistent pricing. Using the valuation formula **(10)**, this time for forward settlement Z_{t+1}, we can write:

$$V_t^n = \frac{1}{R} \left[\mathbb{E}_t [Z_{t+1}] + \frac{\gamma_t}{2} var_t [Z_{t+1}] \right] \tag{14}$$

[4] The number of Offpeak half-hours in a period of time is market specific since it includes public holidays

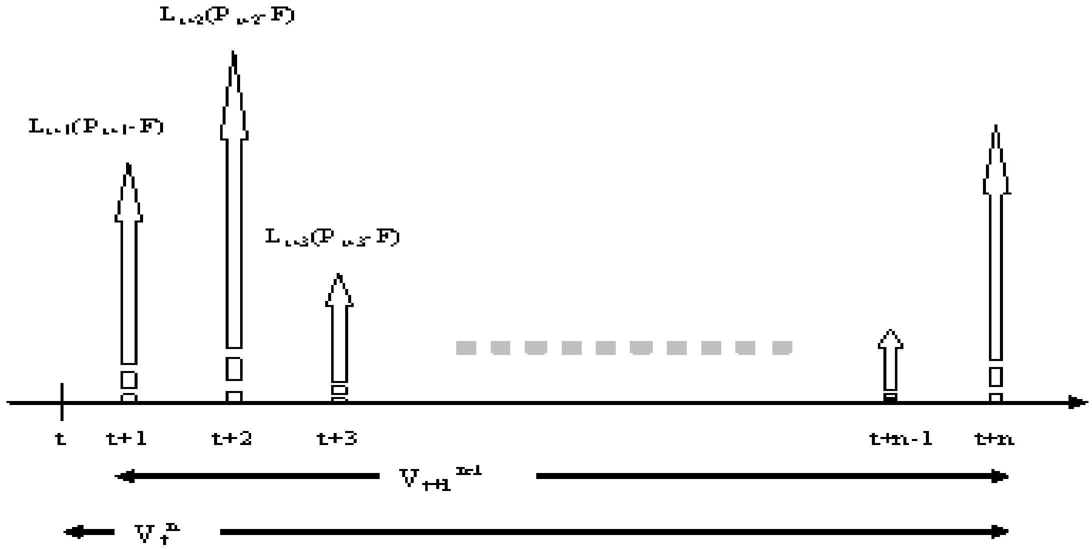

Figure 25: Weekly settlements of a forward contract

where Z_{t+1} is defined in (13). So,

$$\mathbb{E}_t\left[Z_{t+1}\right] = \overline{L}_{t+1}(\mathbb{E}_t\left[P_{t+1}\right] - F) + \mathbb{E}_t\left[V_{t+1}^{n-1}\right] \tag{15}$$

and

$$var_t\left[Z_{t+1}\right] = \overline{L}_{t+1}^2 var_t\left[P_{t+1}\right] + var_t\left[V_{t+1}^{n-1}\right] + 2\overline{L}_{t+1} cov_t(P_{t+1}, V_{t+1}^{n-1}) \tag{16}$$

γ_t is derived earlier from market prices in equation (12) and is reproduced below for clarity of exposition:

$$\gamma_t = \frac{RP_t - \lambda E_t\left[P_{t+1}\right]}{\frac{\lambda^2}{2} var_t\left[P_{t+1}\right]} \tag{17}$$

The system of equations (14), (15), (16) and (17) determines the forward price $F_t(n)$ at time t for a forward contract starting at $t+1$ with maturity n weeks. In order to solve this system, expressions of conditional expectation and variance should be computed. For this purpose, a model for the average price dynamics, like the one developed in section (5), can be considered.

6.3.1 Mathematical Preliminary

In this section, we derive expressions of conditional expectations and variances that are useful in the subsequent section to solve for the forward price F in (Eq.15).

Using the analysis in section (5), we assume that the process of weekly average prices, P_t is such that

$$P_t = e^{s_t + x_t}$$

where $\{s_t\}$ is a deterministic seasonal component, and $\{x_t\}$ is modelled by an $AR(3)$ process

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \phi_3 x_{t-3} + \sigma_t \varepsilon_t$$

where ϕ_i, $i = 1, .., 3$ are the coefficients of the autoregression, $\sigma_t > 0$ is the time-dependent standard deviation of the residuals, and $\varepsilon_t \sim N(0, 1)$ for all $t > 0$.

Lemma 1 *For each time $t > 0$ we have*

$$\begin{aligned} \mathbb{E}_t\left[P_{t+1}\right] &= M_t P_t^{\phi_1} P_{t-1}^{\phi_2} P_{t-2}^{\phi_3} \\ \mathbb{E}_t\left[P_{t+1}^{\theta}\right] &= M(t,\theta) P_t^{\theta\phi_1} P_{t-1}^{\theta\phi_2} P_{t-2}^{\theta\phi_3} \end{aligned}$$

with suitable expressions for M_t and $M(t,\theta)$ which are deterministic.

Proof.

$$\begin{aligned} P_{t+1}^{\theta} &= \exp\left[\theta\left(s_{t+1} + x_{t+1}\right)\right] \\ &= \exp\left[\theta\left(s_{t+1} + \phi_1 x_t + \phi_2 x_{t-1} + \phi_3 x_{t-2} + \sigma_{t+1}\varepsilon_{t+1}\right)\right] \\ &= \exp\left[\begin{array}{c} \theta s_{t+1} - \phi_1\theta s_t - \phi_2\theta s_{t-1} - \phi_3\theta s_{t-2} + \\ \phi_1\theta(x_t + s_t) + \phi_2\theta(x_{t-1} + s_{t-1}) + \\ \phi_3\theta(x_{t-2} + s_{t-2}) + \sigma_{t+1}\theta\varepsilon_{t+1} \end{array}\right] \end{aligned}$$

So the result holds with

$$M(t,\theta) = \exp\left[\theta s_{t+1} - \phi_1\theta s_t - \phi_2\theta s_{t-1} - \phi_3\theta s_{t-2} + \frac{1}{2}\sigma_{t+1}^2\theta^2\right]$$

By notation, $M_t = M(t,1)$. ■

Remark: The random variable $\mathbb{E}_t\left[P_{t+1}\right]$ $\mathcal{F}_t$-measurable. The same derivation above proves that

$$\mathbb{E}_{t+1}\left[P_{t+2}^{\theta}\right] = M(t+1,\theta) P_{t+1}^{\theta\phi_1} P_t^{\theta\phi_2} P_{t-1}^{\theta\phi_3}$$

Lemma 2 *For any real numbers α and β we have*

$$M(t,\alpha)M(t,\beta) = M(t,\alpha+\beta)e^{-\sigma^2\alpha\beta}$$

Proof. straightforward from the properties of exponential and completing the square ■

Lemma 3 (two period-lag) *For each time $t > 0$ we have*

$$\mathbb{E}_t\left[P_{t+2}^{\theta}\right] = M(t+1,\theta)M(t,\theta\phi_1) P_t^{\theta(\phi_1^2+\phi_2)} P_{t-1}^{\theta(\phi_1\phi_2+\phi_3)} P_{t-2}^{\theta\phi_1\phi_3}$$

Proof. Straightforward using Remark (6.3.1) ■

Lemma 4

$$var_t\left[P_{t+1}^{\theta}\right] = N(t,\theta) P_t^{2\theta\phi_1} P_{t-1}^{2\theta\phi_2} P_{t-2}^{2\theta\phi_3}$$

where

$$N(t,\theta) = \left[1 - e^{-\theta^2\sigma^2}\right] M(t,2\theta)$$

Proof.

$$\begin{aligned} var_t\left[P_{t+1}^{\theta}\right] &= \mathbb{E}_t\left[P_{t+1}^{2\theta}\right] - \left(\mathbb{E}_t\left[P_{t+1}^{\theta}\right]\right)^2 \\ &= M(t,2\theta) P_t^{2\theta\phi_1} P_{t-1}^{2\theta\phi_2} P_{t-2}^{2\theta\phi_3} - \left(M(t,\theta) P_t^{\theta\phi_1} P_{t-1}^{\theta\phi_2} P_{t-2}^{\theta\phi_3}\right)^2 \\ &= \left(M(t,2\theta) - M(t,\theta)^2\right) P_t^{2\theta\phi_1} P_{t-1}^{2\theta\phi_2} P_{t-2}^{2\theta\phi_3} \\ &= \left[1 - e^{-\theta^2\sigma^2}\right] M(t,2\theta) P_t^{2\theta\phi_1} P_{t-1}^{2\theta\phi_2} P_{t-2}^{2\theta\phi_3} \end{aligned}$$

■

Lemma 5 *For each time $t > 0$ and for each ν,$\eta > 0$, we have*

$$cov_t\left(P_{t+1}^{\nu}, P_{t+1}^{\eta}\right) = M(t,\nu+\eta)(1 - e^{-\sigma^2\eta\nu}) P_t^{(\nu+\eta)\phi_1} P_{t-1}^{(\nu+\eta)\phi_2} P_{t-2}^{(\nu+\eta)\phi_3}$$

Lemma 6 *If we assume an $AR(1)$ process for P ($\phi_2 = \phi_3 = 0$) then, for each time $t > 0$ and $s > 0$ we have*

$$\mathbb{E}_t\left[P_{t+s}\right] = M(t+s-1,1)M(t+s-2,\phi_1)M(t+s-3,\phi_1^2)...M(t,\phi_1^{s-1})P_t^{\phi_1^s}$$

6.3.2 Week-Ahead Forward Contract (Case $n = 1$)

Applying equation (14) for $n = 1$, we have

$$V_t^1 = \frac{1}{R}\left[\mathbb{E}_t[Z] + \frac{\gamma_t}{2} var_t[Z]\right]$$

where

$$Z = \overline{L}_{t+1}(P_{t+1} - F)$$

This will give a starter value for (14). We have $var_t[Z] = \overline{L}_{t+1}^2 var_t(P_{t+1})$, so, given the expression of γ_t in (17), we have

$$V_t^1 = \frac{1}{R}\left[\overline{L}_{t+1}\left(\mathbb{E}_t[P_{t+1}] - F\right) + \frac{\overline{L}_{t+1}^2}{\lambda^2}\left(RP_t - \lambda E_t[P_{t+1}]\right)\right]$$

And so $F_t(1)$ is the value of F such that $V_t^1 = 0$. Therefore

$$F_t(1) = \mathbb{E}_t[P_{t+1}] + \frac{\overline{L}_{t+1}}{\lambda^2}\left(RP_t - \lambda E_t[P_{t+1}]\right)$$

We note that

$$V_t^1 = a_t^1 P_t + b_t^1 \mathbb{E}_t[P_{t+1}] - c_t^1 F$$

where a_t^1, b_t^1 and c_t^1 are deterministic

$$\begin{cases} a_t^1 = \frac{\overline{L}_{t+1}^2}{\lambda^2} \\ b_t^1 = \frac{\overline{L}_{t+1}}{R} - \frac{\overline{L}_{t+1}^2}{\lambda R} \\ c_t^1 = \frac{\overline{L}_{t+1}}{R} \end{cases}$$

From Lemma (1), the conditional expectation $\mathbb{E}_t[P_{t+1}]$ can be substituted in the expression of V_t^1 so that

$$V_t^1 = a_t^1 P_t + b_t^1 M_t P_t^{\phi_1} P_{t-1}^{\phi_2} P_{t-2}^{\phi_3} - c_t^1 F$$

where M_t, defined in Lemma (1) is function of the seasonal components, the autoregressive coefficients and the volatility in the spot price average.

Therefore, the week-ahead forward price is given by

$$F_t(1) = \frac{1}{c_t^1}\left(a_t^1 P_t + b_t^1 M_t P_t^{\phi_1} P_{t-1}^{\phi_2} P_{t-2}^{\phi_3}\right)$$

which is a function of current and past average prices, contract volume for the period contracted, the market price of risk and interest rates.

6.3.3 General Case: n-Period Forward Contract

Applying (14) to $(n+1)$-period contract, we have

$$V_t^{n+1} = \frac{1}{R}\left[\mathbb{E}_t[Z_{t+1}] + \frac{\gamma_t}{2} var_t[Z_{t+1}]\right]$$

where

$$\begin{aligned} Z_{t+1} &= \overline{L}_{t+1}(P_{t+1} - F) + V_{t+1}^n \\ &= \overline{L}_{t+1}(P_{t+1} - F) + \sum_k a_{t+1}^n(k) P_{t+1}^{\alpha_k^n} P_t^{\beta_k^n} P_{t-1}^{\gamma_k^n} - c_{t+1}^n F \end{aligned}$$

Let us calculate the expectation and the variance in the expression of V_t^{n+1}.

First, the expectation

$$\begin{aligned} \mathbb{E}_t[Z_{t+1}] &= \overline{L}_{t+1}(\mathbb{E}_t[P_{t+1}] - F) + \\ &\quad \sum_k a_{t+1}^n(k) P_t^{\beta_k^n} P_{t-1}^{\gamma_k^n} \mathbb{E}_t\left[P_{t+1}^{\alpha_k^n}\right] - c_{t+1}^n F \end{aligned}$$

We can already deduce

$$\begin{aligned} c_t^{n+1} &= \frac{\overline{L}_{t+1} + c_{t+1}^n}{R} \\ &= \frac{1}{R}\overline{L}_{t+1} + \frac{1}{R^2}\overline{L}_{t+2} + ... + \frac{1}{R^{n+1}}\overline{L}_{t+n+1} \end{aligned}$$

We can prove by induction that the price $F_t(n)$ is given by

$$F_t(n) = \frac{1}{c_t^n}\sum_{k=1}^{n} a_t^n(k) P_t^{\alpha_k^n} P_{t-1}^{\beta_k^n} P_{t-2}^{\gamma_k^n} \tag{18}$$

where $a_t^n(k)$, α_k^n, β_k^n and γ_k^n are deterministic coefficients, computed in a recursive fashion.

6.4 Discussion of the Results

By examining the expression (18) of the forward price, we note the following points. The forward price at time t for n periods ahead, $F_t(n)$, is a function of spot price averages for the current period, P_t as well as for the past two periods, P_{t-1} and P_{t-2}. This is a consequence of the AR(3) model assumed for the spot price average. Moreover, the denominator c_t^n depends on the volumes contracted for each period in the future, $\overline{L}_{t+i}$, $1 \leq i \leq n+1$, and the interest rate R,and the market price of risk through the coefficients $a_t^n(k)$[5]. This feature of the model makes it more realistic than the cost of carry type models where no volume risk premium is embedded in the forward price.

It should be stressed that the normality assumption for the pool price average is not too unrealistic. This can be justified by the central limit theorem and the calculated p-value for the Jarque-Bera test of 9.72%. The assumptions of the central limit theorem are not exactly verified in this case. Even though there are 7x48 = 336 observations to be averaged, the independence assumption does not hold. The autocorrelation structure depicted in Figure (22) shows a non-negligeable serial correlation in the residuals.

The normality assumption can be relaxed and a better fit for the residuals using distributions from the elliptical contoured class can be done. The new distribution should be chosen carefully in order to maintain a derivation of a tractable solution.

Nevertheless, solving the forward pricing problem using the real option theory, as outlined in this paper, can be considered as a first step in a new paradigm of forward pricing.

7 Conclusion

This paper introduces a novel approach to pricing electricity forwards based on real options theory. It starts by investigating the risk faced by the electricity market players and the role that forward contracts play in mitigating this risk. Then it examines the characteristics of electricity pool prices and models the price average dynamics using autoregressive models for peak and off-peak partitions of the day. Forward prices are then derived using the dynamics of the pool price average and a generalised concept of certainty equivalent, introduced in the presence of trading portfolios.

Unlike most of the literature about forward pricing, the resulting forward prices depend on the volume of the underlying contract. This feature of the model makes it more realistic as it accounts for the volume risk premium. While the normality assumption provides a tractable solution for the forward price, it should be changed to a more accurate modelling of the residuals of the price average. Further research consists of calibrating the model to market data and comparing its performance to existing forward pricing models.

[5] See the expression of $F_t(1)$ in section (6.3.2).

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